Using Pysketcher to Create Principal Sketches of Physics Problems

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Abstract

Pysketcher is a Python package which allows principal sketches of physics and mechanics problems to be realized through short programs instead of interactive (and potentially tedious and inaccurate) drawing. Elements of the sketch, such as lines, circles, angles, forces, coordinate systems, etc., are realized as objects and collected in hierarchical structures. Parts of the hierarchical structures can easily change line styles and colors, or be copied, scaled, translated, and rotated. These features make it straightforward to move parts of the sketch to create animation, usually in accordance with the physics of the underlying problem. Exact dimensioning of the elements in the sketch is trivial to obtain since distances are specified in computer code.

Pysketcher is easy to learn from a number of examples. Beyond essential Python programming and a knowledge about mechanics problems, no further background is required.

Contents

1 A first glimpse of Pysketcher

Formulation of physical problems makes heavy use of *principal sketches* such as the one in Figure 1. This particular sketch illustrates the classical mechanics problem of a rolling wheel on an inclined plane. The figure is made up many individual elements: a rectangle filled with a pattern (the inclined plane), a hollow circle with color (the wheel), arrows with labels (the N and Mg forces, and the x axis), an angle with symbol θ, and a dashed line indicating the starting location of the wheel.

Drawing software and plotting programs can produce such figures quite easily in principle, but the amount of details the user needs to control with the mouse can be substantial. Software more tailored to producing sketches of this type would work with more convenient abstractions, such as circle, wall, angle, force arrow, axis, and so forth. And as soon we start *programming* to construct the figure we get a range of other powerful tools at disposal. For example, we can easily translate and rotate parts of the figure and make an animation that illustrates the physics of the problem. Programming as a superior alternative to interactive drawing is the mantra of this section.

1.1 Basic construction of sketches

Before attacking real-life sketches as in Figure 1 we focus on the significantly simpler drawing shown in Figure 2. This toy sketch consists of several elements: two circles, two rectangles, and a “ground” element.

When the sketch is defined in terms of computer code, it is natural to parameterize geometric features, such as the radius of the wheel (R), the center point of the left wheel (w_1), as well as the height (H) and length (L) of the main part. The simple vehicle in Figure 2 is quickly drawn in almost any interactive tool. However, if we want to change the radius of the wheels, you

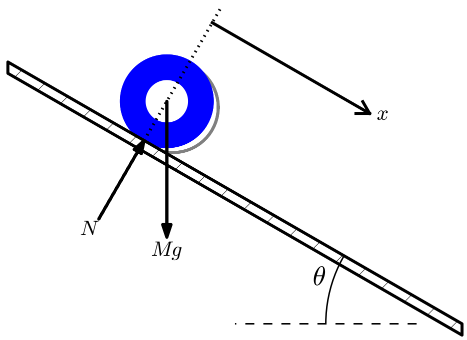


Figure 1: Sketch of a physics problem.

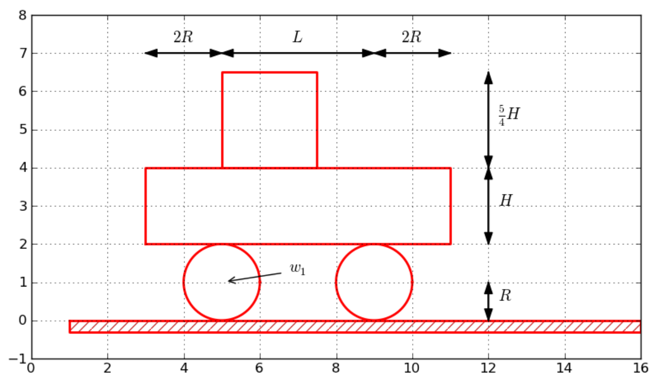


Figure 2: Sketch of a simple figure.

need a sophisticated drawing tool to avoid redrawing the whole figure, while in computer code this is a matter of changing the R parameter and rerunning the program. For example, Figure 3 shows a variation of the drawing in Figure 2 obtained by just setting $R = 0.5$, $L = 5$, $H = 2$, and $R = 2$. Being able to quickly change geometric sizes is key to many problem settings in physics and engineering, but then a program must define the geometry.

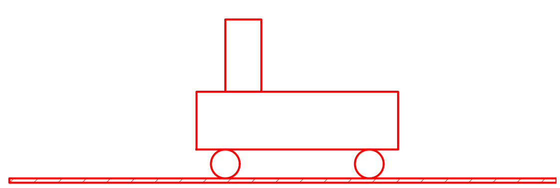


Figure 3: Redrawing a figure with other geometric parameters.

Basic drawing. A typical program creating these five elements is shown next. After importing the `pysketcher` package, the first task is always to define a coordinate system:

```
from pysketcher import *

drawing_tool.set_coordinate_system(
    xmin=0, xmax=10, ymin=-1, ymax=8)
```

Instead of working with lengths expressed by specific numbers it is highly recommended to use variables to parameterize lengths as this makes it easier to change dimensions later. Here we introduce some key lengths for the radius of the wheels, distance between the wheels, etc.:

```
R = 1    # radius of wheel
L = 4    # distance between wheels
H = 2    # height of vehicle body
w_1 = 5  # position of front wheel
drawing_tool.set_coordinate_system(xmin=0, xmax=w_1 + 2*L + 3*R,
                                   ymin=-1, ymax=2*R + 3*H)
```

With the drawing area in place we can make the first `Circle` object in an intuitive fashion:

```
wheel1 = Circle(center=(w_1, R), radius=R)
```

to change dimensions later.

To translate the geometric information about the `wheel1` object to instructions for the plotting engine (in this case Matplotlib), one calls the `wheel1.draw()`. To display all drawn objects, one issues `drawing_tool.display()`. The typical steps are hence:

```
wheel1 = Circle(center=(w_1, R), radius=R)
wheel1.draw()

# Define other objects and call their draw() methods
drawing_tool.display()
drawing_tool.savefig('tmp.png')  # store picture
```

The next wheel can be made by taking a copy of `wheel1` and translating the object to the right according to a displacement vector $(L, 0)$:

```
wheel2 = wheel1.copy()
wheel2.translate((L,0))
```

The two rectangles are also made in an intuitive way:

```
under = Rectangle(lower_left_corner=(w_1-2*R, 2*R),
                  width=2*R + L + 2*R, height=H)
over  = Rectangle(lower_left_corner=(w_1, 2*R + H),
                  width=2.5*R, height=1.25*H)
```

Groups of objects. Instead of calling the `draw` method of every object, we can group objects and call `draw`, or perform other operations, for the whole group. For example, we may collect the two wheels in a `wheels` group and the `over` and `under` rectangles in a `body` group. The whole vehicle is a composition of its `wheels` and `body` groups. The code goes like

```
wheels  = Composition({'wheel1': wheel1, 'wheel2': wheel2})
body    = Composition({'under': under, 'over': over})

vehicle = Composition({'wheels': wheels, 'body': body})
```

The ground is illustrated by an object of type `Wall`, mostly used to indicate walls in sketches of mechanical systems. A `Wall` takes the `x` and `y` coordinates of some curve, and a `thickness` parameter, and creates a thick curve filled with a simple pattern. In this case the curve is just a flat line so the construction is made of two points on the ground line ($(w_1 - L, 0)$ and $(w_1 + 3L, 0)$):

```
ground = Wall(x=[w_1 - L, w_1 + 3*L], y=[0, 0], thickness=-0.3*R)
```

The negative thickness makes the pattern-filled rectangle appear below the defined line, otherwise it appears above.

We may now collect all the objects in a "top" object that contains the whole figure:

```
fig = Composition({'vehicle': vehicle, 'ground': ground})
fig.draw()  # send all figures to plotting backend
drawing_tool.display()
drawing_tool.savefig('tmp.png')
```

The `fig.draw()` call will visit all subgroups, their subgroups, and so forth in the hierarchical tree structure of figure elements, and call `draw` for every object.

Changing line styles and colors. Controlling the line style, line color, and line width is fundamental when designing figures. The `pysketcher` package allows the user to control such properties in single objects, but also set global properties that are used if the object has no particular specification of the properties. Setting the global properties are done like

```
drawing_tool.set_linestyle('dashed')
drawing_tool.set_linecolor('black')
drawing_tool.set_linewidth(4)
```

At the object level the properties are specified in a similar way:

```
wheels.set_linestyle('solid')
wheels.set_linecolor('red')
```

and so on.

Geometric figures can be specified as *filled*, either with a color or with a special visual pattern:

```
# Set filling of all curves
drawing_tool.set_filled_curves(color='blue', pattern='/')
```

```
# Turn off filling of all curves
drawing_tool.set_filled_curves(False)

# Fill the wheel with red color
wheel1.set_filled_curves('red')
```

The figure composition as an object hierarchy. The composition of objects making up the figure is hierarchical, similar to a family, where each object has a parent and a number of children. Do a `print fig` to display the relations:

```
ground
    wall
vehicle
    body
        over
            rectangle
        under
            rectangle
    wheels
        wheel1
            arc
        wheel2
            arc
```

The indentation reflects how deep down in the hierarchy (family) we are. This output is to be interpreted as follows:

- `fig` contains two objects, `ground` and `vehicle`
- `ground` contains an object `wall`
- `vehicle` contains two objects, `body` and `wheels`
- `body` contains two objects, `over` and `under`
- `wheels` contains two objects, `wheel1` and `wheel2`

In this listing there are also objects not defined by the programmer: `rectangle` and `arc`. These are of type `Curve` and automatically generated by the classes `Rectangle` and `Circle`.

More detailed information can be printed by

```
print fig.show_hierarchy('std')
```

yielding the output

```
ground (Wall):
    wall (Curve): 4 coords fillcolor='white' fillpattern='/'
vehicle (Composition):
    body (Composition):
        over (Rectangle):
            rectangle (Curve): 5 coords
        under (Rectangle):
            rectangle (Curve): 5 coords
    wheels (Composition):
        wheel1 (Circle):
            arc (Curve): 181 coords
        wheel2 (Circle):
            arc (Curve): 181 coords
```

Here we can see the class type for each figure object, how many coordinates that are involved in basic figures (`Curve` objects), and special settings of the basic figure (fillcolor, line types, etc.). For example, `wheel2` is a `Circle` object consisting of an `arc`, which is a `Curve` object consisting of 181 coordinates (the points needed to draw a smooth circle). The `Curve` objects are the only objects that really holds specific coordinates to be drawn. The other object types are just compositions used to group parts of the complete figure.

One can also get a graphical overview of the hierarchy of figure objects that build up a particular figure `fig`. Just call `fig.graphviz_dot('fig')` to produce a file `fig.dot` in the *dot format*. This file contains relations between parent and child objects in the figure and can be turned into an image, as in Figure 4, by running the `dot` program:

```
Terminal> dot -Tpng -o fig.png fig.dot
```

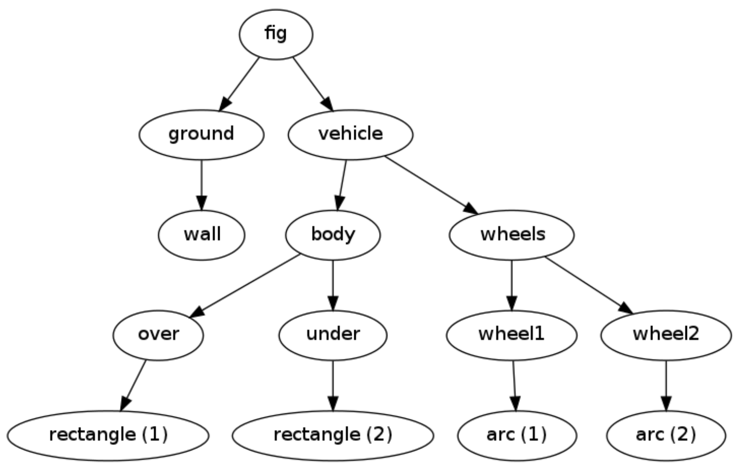


Figure 4: Hierarchical relation between figure objects.

The call `fig.graphviz_dot('fig', classname=True)` makes a `fig.dot` file where the class type of each object is also visible, see Figure 5. The ability to write out the object hierarchy or view it graphically can be of great help when working with complex figures that involve layers of subfigures.

Any of the objects can in the program be reached through their names, e.g.,

```
fig['vehicle']
fig['vehicle']['wheels']
fig['vehicle']['wheels']['wheel2']
fig['vehicle']['wheels']['wheel2']['arc']
fig['vehicle']['wheels']['wheel2']['arc'].x  # x coords
fig['vehicle']['wheels']['wheel2']['arc'].y  # y coords
fig['vehicle']['wheels']['wheel2']['arc'].linestyle
fig['vehicle']['wheels']['wheel2']['arc'].linetype
```

Grabbing a part of the figure this way is handy for changing properties of that part, for example, colors, line styles (see Figure 6):

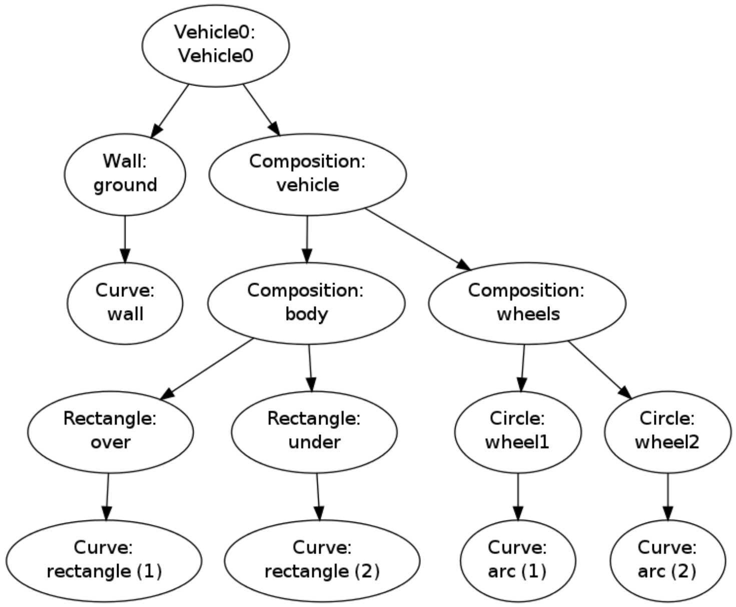


Figure 5: Hierarchical relation between figure objects, including their class names.

```
fig['vehicle']['wheels'].set_filled_curves('blue')
fig['vehicle']['wheels'].set_linewidth(6)
fig['vehicle']['wheels'].set_linecolor('black')

fig['vehicle']['body']['under'].set_filled_curves('red')

fig['vehicle']['body']['over'].set_filled_curves(pattern='/')
fig['vehicle']['body']['over'].set_linewidth(14)
fig['vehicle']['body']['over']['rectangle'].linewidth = 4
```

The last line accesses the `Curve` object directly, while the line above, accesses the `Rectangle` object, which will then set the linewidth of its `Curve` object, and other objects if it had any. The result of the actions above is shown in Figure 6.

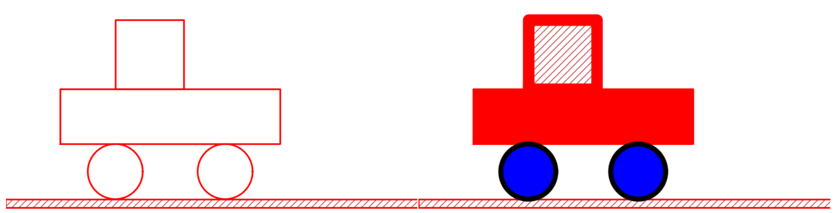


Figure 6: Left: Basic line-based drawing. Right: Thicker lines and filled parts.

We can also change position of parts of the figure and thereby make animations, as shown next.

Animation: translating the vehicle. Can we make our little vehicle roll? A first attempt will be to fake rolling by just displacing all parts of the vehicle. The relevant parts constitute the `fig['vehicle']` object. This part of the figure can be translated, rotated, and scaled. A translation along the ground means a translation in x direction, say a length L to the right:

```
fig['vehicle'].translate((L,0))
```

You need to erase, draw, and display to see the movement:

```
drawing_tool.erase()
fig.draw()
drawing_tool.display()
```

Without erasing, the old drawing of the vehicle will remain in the figure so you get two vehicles. Without `fig.draw()` the new coordinates of the vehicle will not be communicated to the drawing tool, and without calling display the updated drawing will not be visible.

A figure that moves in time is conveniently realized by the function `animate`:

```
animate(fig, tp, action)
```

Here, `fig` is the entire figure, `tp` is an array of time points, and `action` is a user-specified function that changes `fig` at a specific time point. Typically, `action` will move parts of `fig`.

In the present case we can define the movement through a velocity function `v(t)` and displace the figure `v(t)*dt` for small time intervals `dt`. A possible velocity function is

```
def v(t):
    return -8*R*t*(1 - t/(2*R))
```

Our action function for horizontal displacements `v(t)*dt` becomes

```
def move(t, fig):
    x_displacement = dt*v(t)
    fig['vehicle'].translate((x_displacement, 0))
```

Since our velocity is negative for $t \in [0, 2R]$ the displacement is to the left.

The `animate` function will for each time point `t` in `tp` erase the drawing, call `action(t, fig)`, and show the new figure by `fig.draw()` and `drawing_tool.display()`. Here we choose a resolution of the animation corresponding to 25 time points in the time interval $[0, 2R]$:

```
import numpy
tp = numpy.linspace(0, 2*R, 25)
dt = tp[1] - tp[0]  # time step

animate(fig, tp, move, pause_per_frame=0.2)
```

The `pause_per_frame` adds a pause, here 0.2 seconds, between each frame in the animation.

We can also ask `animate` to store each frame in a file:

```
files = animate(fig, tp, move_vehicle, moviefiles=True,
                pause_per_frame=0.2)
```

The `files` variable, here `'tmp_frame_%04d.png'`, is the printf-specification used to generate the individual plot files. We can use this specification to make a video file via `ffmpeg` (or `avconv` on Debian-based Linux systems such as Ubuntu). Videos in the Flash and WebM formats can be created by

```
Terminal> ffmpeg -r 12 -i tmp_frame_%04d.png -vcodec flv mov.flv
Terminal> ffmpeg -r 12 -i tmp_frame_%04d.png -vcodec libvpx mov.webm
```

An animated GIF movie can also be made using the `convert` program from the ImageMagick software suite:

```
Terminal> convert -delay 20 tmp_frame*.png mov.gif
Terminal> animate mov.gif  # play movie
```

The delay between frames, in units of 1/100 s, governs the speed of the movie. To play the animated GIF file in a web page, simply insert `<img src="mov.gif">` in the HTML code.

The individual PNG frames can be directly played in a web browser by running

```
Terminal> scitools movie output_file=mov.html fps=5 tmp_frame*
```

or calling

```
from scitools.std import movie
movie(files, encoder='html', output_file='mov.html')
```

in Python. Load the resulting file `mov.html` into a web browser to play the movie.

Try to run `vehicle0.py` and then load `mov.html` into a browser, or play one of the `mov.*` video files. Alternatively, you can view a ready-made movie.

Animation: rolling the wheels. It is time to show rolling wheels. To this end, we add spokes to the wheels, formed by two crossing lines, see Figure 7. The construction of the wheels will now involve a circle and two lines:

```
wheel1 = Composition({
    'wheel': Circle(center=(w_1, R), radius=R),
    'cross': Composition({'cross1': Line((w_1,0),   (w_1,2*R)),
                          'cross2': Line((w_1-R,R), (w_1+R,R))})})
wheel2 = wheel1.copy()
wheel2.translate((L,0))
```

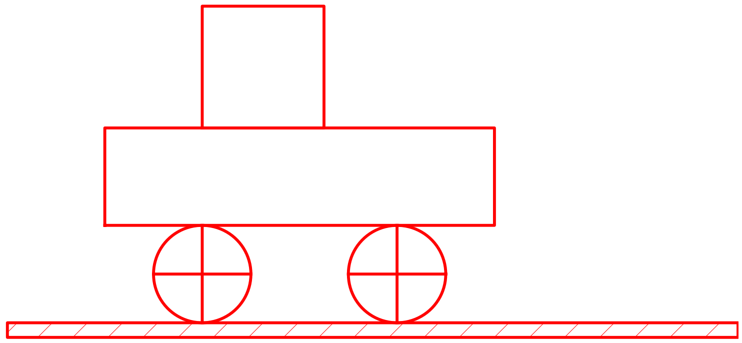


Figure 7: Wheels with spokes to illustrate rolling.

Observe that `wheel1.copy()` copies all the objects that make up the first wheel, and `wheel2.translate` translates all the copied objects.

The `move` function now needs to displace all the objects in the entire vehicle and also rotate the `cross1` and `cross2` objects in both wheels. The rotation angle follows from the fact that the arc length of a rolling wheel equals the displacement of the center of the wheel, leading to a rotation angle

```
angle = - x_displacement/R
```

With `w_1` tracking the x coordinate of the center of the front wheel, we can rotate that wheel by

```
w1 = fig['vehicle']['wheels']['wheel1']
from math import degrees
w1.rotate(degrees(angle), center=(w_1, R))
```

The `rotate` function takes two parameters: the rotation angle (in degrees) and the center point of the rotation, which is the center of the wheel in this case. The other wheel is rotated by

```
w2 = fig['vehicle']['wheels']['wheel2']
w2.rotate(degrees(angle), center=(w_1 + L, R))
```

That is, the angle is the same, but the rotation point is different. The update of the center point is done by `w_1 += x_displacement`. The complete `move` function with translation of the entire vehicle and rotation of the wheels then becomes

```
w_1 = w_1 + L   # start position

def move(t, fig):
    x_displacement = dt*v(t)
    fig['vehicle'].translate((x_displacement, 0))

    # Rotate wheels
    global w_1
    w_1 += x_displacement
    # R*angle = -x_displacement
```

```
angle = - x_displacement/R
w1 = fig['vehicle']['wheels']['wheel1']
w1.rotate(degrees(angle), center=(w_1, R))
w2 = fig['vehicle']['wheels']['wheel2']
w2.rotate(degrees(angle), center=(w_1 + L, R))
```

The complete example is found in the file `vehicle1.py`. You may run this file or watch a ready-made movie.

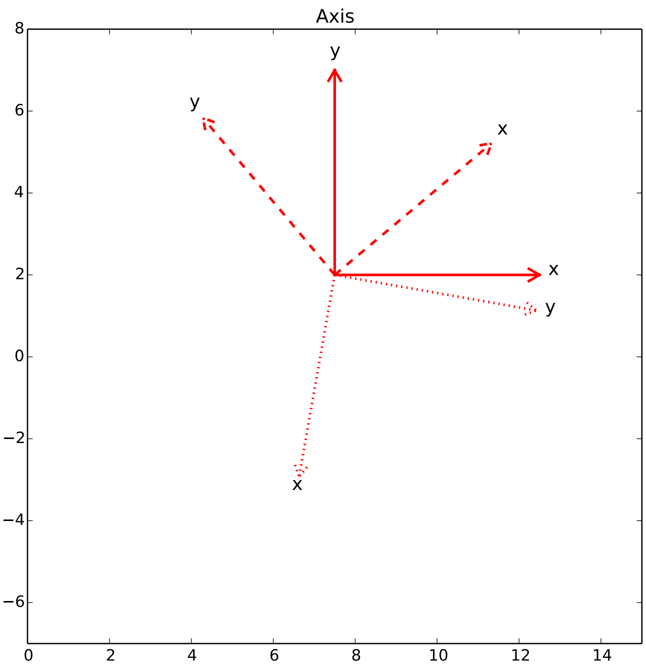
The advantages with making figures this way, through programming rather than using interactive drawing programs, are numerous. For example, the objects are parameterized by variables so that various dimensions can easily be changed. Subparts of the figure, possible involving a lot of figure objects, can change color, linetype, filling or other properties through a *single* function call. Subparts of the figure can be rotated, translated, or scaled. Subparts of the figure can also be copied and moved to other parts of the drawing area. However, the single most important feature is probably the ability to make animations governed by mathematical formulas or data coming from physics simulations of the problem, as shown in the example above.

2 Basic shapes

This section presents many of the basic shapes in Pysketcher: `Axis`, `Distance_wText`, `Rectangle`, `Triangle`, `Arc`, `Spring`, `Dashpot`, and `Wavy`. Each shape is demonstrated with a figure and a unit test that shows how the figure is constructed in Python code. These demos rely heavily on the method `draw_dimensions` in the shape classes, which annotates the basic drawing of the shape with the various geometric parameters that govern the shape.

2.1 Axis

The `Axis` object gives the possibility draw a single axis to notify a coordinate system. Here is an example where we draw x and y axis of three coordinate systems of different rotation:



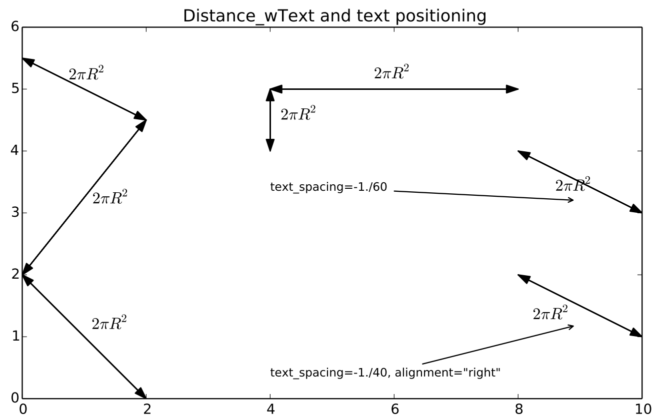
The corresponding code looks like this:

```
def test_Axis():
    drawing_tool.set_coordinate_system(
        xmin=0, xmax=15, ymin=-7, ymax=8, axis=True,
        instruction_file='tmp_Axis.py')
    # Draw normal x and y axis with origin at (7.5, 2)
    # in the coordinate system of the sketch: [0,15]x[-7,8]
    x_axis = Axis((7.5,2), 5, 'x', rotation_angle=0)
    y_axis = Axis((7.5,2), 5, 'y', rotation_angle=90)
    system = Composition({'x axis': x_axis, 'y axis': y_axis})
    system.draw()
    drawing_tool.display()

    # Rotate this system 40 degrees counter clockwise
    # and draw it with dashed lines
    system.set_linestyle('dashed')
    system.rotate(40, (7.5,2))
    system.draw()
    drawing_tool.display()

    # Rotate this system another 220 degrees and show
    # with dotted lines
    system.set_linestyle('dotted')
    system.rotate(220, (7.5,2))
    system.draw()
    drawing_tool.display()

    drawing_tool.display('Axis')
```

2.2 Distance with text

The object `Distance_wText` is used to display an arrow, to indicate a distance in a sketch, with an additional text in the middle of the arrow.

The figure

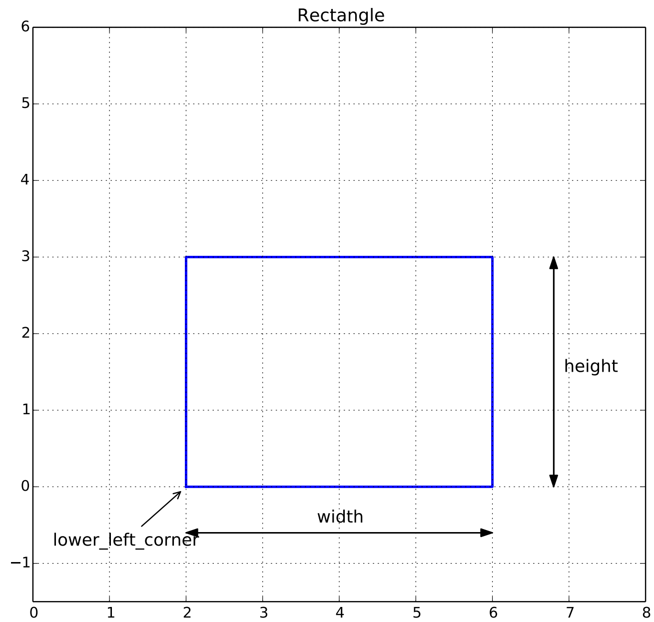


was produced by this code:

```
def test_Distance_wText():
    drawing_tool.set_coordinate_system(
        xmin=0, xmax=10, ymin=0, ymax=6,
        axis=True, instruction_file='tmp_Distance_wText.py')

    fontsize=14
    t = r'$ 2\pi R^2 $'  # sample text
    examples = Composition({
        'a0': Distance_wText((4,5), (8, 5), t, fontsize),
        'a6': Distance_wText((4,5), (4, 4), t, fontsize),
        'a1': Distance_wText((0,2), (2, 4.5), t, fontsize),
        'a2': Distance_wText((0,2), (2, 0), t, fontsize),
        'a3': Distance_wText((2,4.5), (0, 5.5), t, fontsize),
        'a4': Distance_wText((8,4), (10, 3), t, fontsize,
                             text_spacing=old_div(-1.,60)),
        'a5': Distance_wText((8,2), (10, 1), t, fontsize,
                             text_spacing=old_div(-1.,40), alignment='right'),
        'c1': Text_wArrow('text_spacing=-1./60',
                          (4, 3.5), (9, 3.2),
                          fontsize=10, alignment='left'),
        'c2': Text_wArrow('text_spacing=-1./40, alignment="right"',
                          (4, 0.5), (9, 1.2),
                          fontsize=10, alignment='left'),
        })
    examples.draw()
    drawing_tool.display('Distance_wText and text positioning')
```

Note the use of `Text_wArrow` to write an explaining text with an associated arrow, here used to explain how the `text_spacing` and `alignment` arguments can be used to adjust the appearance of the text that goes with the distance arrow.

2.3 Rectangle

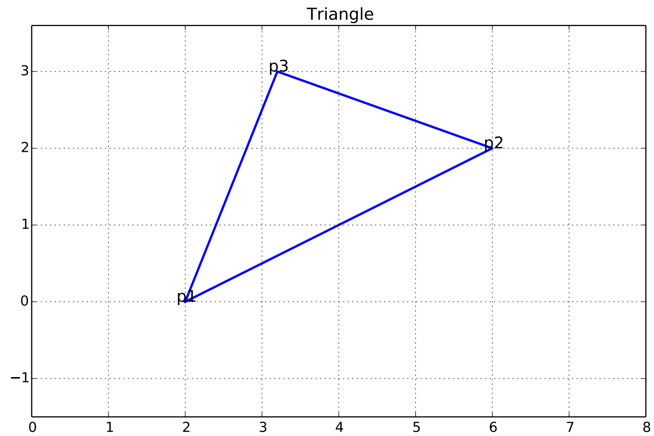


The above figure can be produced by the following code.

```
def test_Rectangle():
    L = 3.0
    W = 4.0

    drawing_tool.set_coordinate_system(
        xmin=0, xmax=2*W, ymin=old_div(-L,2), ymax=2*L,
        axis=True, instruction_file='tmp_Rectangle.py')
    drawing_tool.set_linecolor('blue')
    drawing_tool.set_grid(True)

    xpos = old_div(W,2)
    r = Rectangle(lower_left_corner=(xpos,0), width=W, height=L)
    r.draw()
    r.draw_dimensions()
    drawing_tool.display('Rectangle')
```

Note that the `draw_dimension` method adds explanation of dimensions and various important argument in the construction of a shape. It adapts the annotations to the geometry of the current shape.

2.4 Triangle

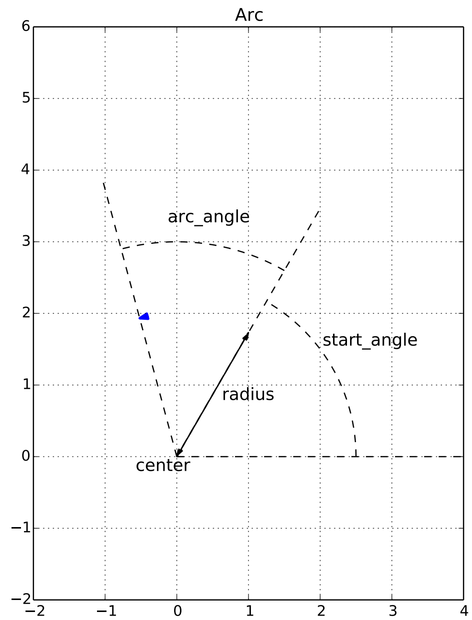


The code below produces the figure.

```
def test_Triangle():
    L = 3.0
    W = 4.0

    drawing_tool.set_coordinate_system(
        xmin=0, xmax=2*W, ymin=old_div(-L,2), ymax=1.2*L,
        axis=True, instruction_file='tmp_Triangle.py')
    drawing_tool.set_linecolor('blue')
    drawing_tool.set_grid(True)

    xpos = 1
    t = Triangle(p1=(old_div(W,2),0), p2=(3*W/2,old_div(W,2)), p3=(4*W/5.,L))
    t.draw()
    t.draw_dimensions()
    drawing_tool.display('Triangle')
```

Here, the `draw_dimension` method writes the name of the corners at the position of the corners, which does not always look nice (the present figure is an example). For a high-quality sketch one would add some spacing to the location of the p1, p2, and even p3 texts.

2.5 Arc

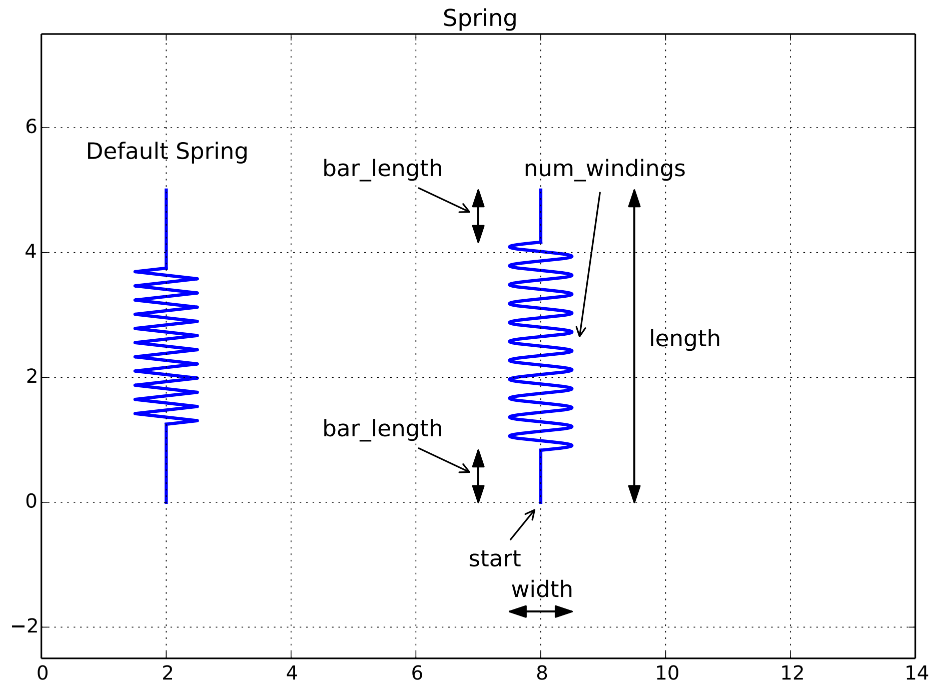


An arc like the one above is produced by

```
def test_Arc():
    L = 4.0
    W = 4.0

    drawing_tool.set_coordinate_system(
        xmin=old_div(-W,2), xmax=W, ymin=old_div(-L,2), ymax=1.5*L,
        axis=True, instruction_file='tmp_Arc.py')
    drawing_tool.set_linecolor('blue')
    drawing_tool.set_grid(True)

    center = point(0,0)
    radius = old_div(L,2)
    start_angle = 60
    arc_angle = 45
    a = Arc(center, radius, start_angle, arc_angle)
    a.draw()

    R1 = 1.25*radius
    R2 = 1.5*radius
    R = 2*radius
    a.dimensions = {
        'start_angle':
        Arc_wText(
            'start_angle', center, R1, start_angle=0,
            arc_angle=start_angle, text_spacing=old_div(1,10.)),
        'arc_angle':
        Arc_wText(
            'arc_angle', center, R2, start_angle=start_angle,
            arc_angle=arc_angle, text_spacing=old_div(1,20.)),
```

```
        'r=0':
        Line(center, center +
             point(R*cos(radians(start_angle)),
                   R*sin(radians(start_angle)))),
        'r=start_angle':
        Line(center, center +
             point(R*cos(radians(start_angle+arc_angle)),
                   R*sin(radians(start_angle+arc_angle)))),
        'r=start+arc_angle':
        Line(center, center +
             point(R, 0)).set_linestyle('dashed'),
        'radius': Distance_wText(center, a(0), 'radius', text_spacing=old_div(1,40.)),
        'center': Text('center', center-point(old_div(radius,10.), old_div(radius,10.))),
        }
    for dimension in a.dimensions:
        if dimension.startswith('r='):
            dim = a.dimensions[dimension]
            dim.set_linestyle('dashed')
            dim.set_linewidth(1)
            dim.set_linecolor('black')

    a.draw_dimensions()
    drawing_tool.display('Arc')
```

2.6 Spring

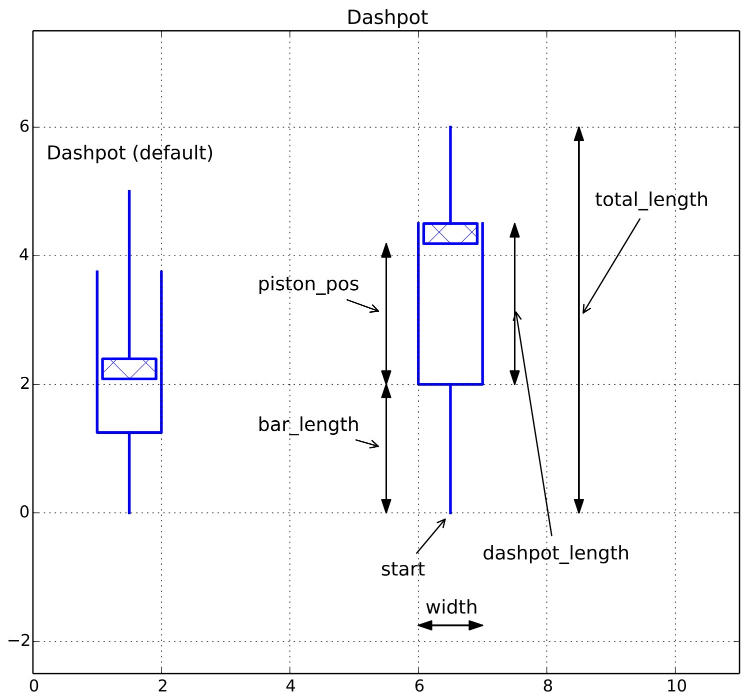


The code for making these two springs goes like this:

```
def test_Spring():
    L = 5.0
```

```
    W = 2.0

    drawing_tool.set_coordinate_system(
        xmin=0, xmax=7*W, ymin=old_div(-L,2), ymax=1.5*L,
        axis=True, instruction_file='tmp_Spring.py')
    drawing_tool.set_linecolor('blue')
    drawing_tool.set_grid(True)

    xpos = W
    s1 = Spring((W,0), L, teeth=True)
    s1_title = Text('Default Spring',
                    s1.geometric_features()['end'] + point(0,old_div(L,10)))
    s1.draw()
    s1_title.draw()
    #s1.draw_dimensions()
    xpos += 3*W
    s2 = Spring(start=(xpos,0), length=L, width=old_div(W,2.),
                bar_length=old_div(L,6.), teeth=False)
    s2.draw()
    s2.draw_dimensions()
    drawing_tool.display('Spring')
```

2.7 Dashpot



This dashpot is produced by

```
def test_Dashpot():
    L = 5.0
    W = 2.0
```

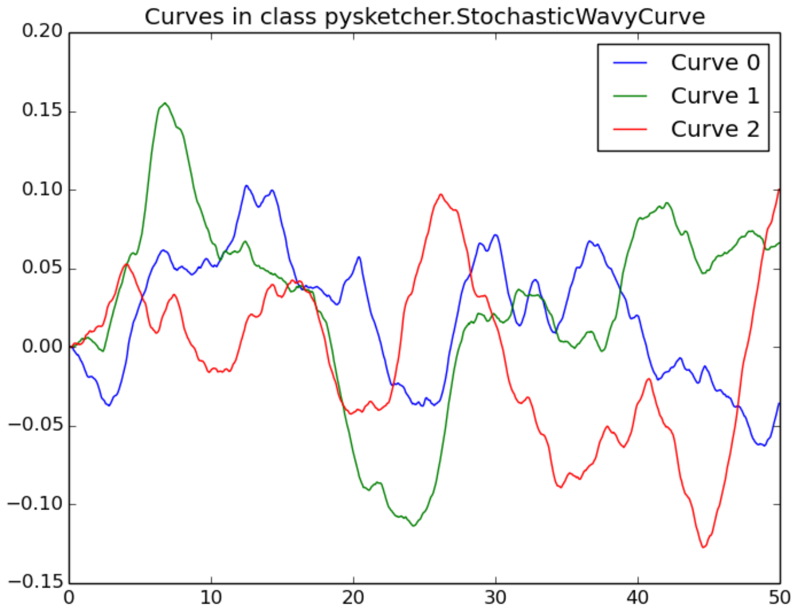
```
xpos = 0

drawing_tool.set_coordinate_system(
    xmin=xpos, xmax=xpos+5.5*W, ymin=old_div(-L,2), ymax=1.5*L,
    axis=True, instruction_file='tmp_Dashpot.py')
drawing_tool.set_linecolor('blue')
drawing_tool.set_grid(True)

# Default (simple) dashpot
xpos = 1.5
d1 = Dashpot(start=(xpos,0), total_length=L)
d1_title = Text('Dashpot (default)',
                d1.geometric_features()['end'] + point(0,old_div(L,10)))
d1.draw()
d1_title.draw()

# Dashpot for animation with fixed bar_length, dashpot_length and
# prescribed piston_pos
xpos += 2.5*W
d2 = Dashpot(start=(xpos,0), total_length=1.2*L, width=old_div(W,2),
             bar_length=W, dashpot_length=old_div(L,2), piston_pos=2*W)
d2.draw()
d2.draw_dimensions()

drawing_tool.display('Dashpot')
```

2.8 Wavy

Looks strange. Fix x axis.

2.9 Stochastic curves

The `StochasticWavyCurve` object offers three precomputed graphics that have a random variation:



The usage is simple. The construction

```
curve = StochasticWavyCurve(curve_no=1, percentage=40)
```

picks the second curve (the three are numbered 0, 1, and 2), and the first 40% of that curve. In case one desires another extent of the axis, one can just scale the coordinates directly as these are stored in the arrays `curve.x[curve_no]` and `curve.y[curve_no]`.

3 Inner workings of the Pysketcher tool

We shall now explain how we can, quite easily, realize software with the capabilities demonstrated in the previous examples. Each object in the figure is represented as a class in a class hierarchy. Using inheritance, classes can inherit properties from parent classes and add new geometric features.

Class programming is a key technology for realizing Pysketcher. As soon as some classes are established, more are easily added. Enhanced functionality for all the classes is also easy to implement in common, generic code that can immediately be shared by all present and future classes. The fundamental data structure involved in the `pysketcher` package is a hierarchical tree, and much of the material on implementation issues targets how to traverse tree structures with recursive function calls in object hierarchies. This topic is of key relevance in a wide range of other applications as well. In total, the inner workings of Pysketcher constitute an excellent example on the power of class programming.

3.1 Example of classes for geometric objects

We introduce class `Shape` as superclass for all specialized objects in a figure. This class does not store any data, but provides a series of functions that add functionality to all the subclasses. This will be shown later.

Simple geometric objects. One simple subclass is `Rectangle`, specified by the coordinates of the lower left corner and its width and height:

```
class Rectangle(Shape):
    def __init__(self, lower_left_corner, width, height):
        p = lower_left_corner  # short form
        x = [p[0], p[0] + width,
             p[0] + width, p[0], p[0]]
        y = [p[1], p[1], p[1] + height,
             p[1] + height, p[1]]
        self.shapes = {'rectangle': Curve(x,y)}
```

Any subclass of `Shape` will have a constructor that takes geometric information about the shape of the object and creates a dictionary `self.shapes` with the shape built of simpler shapes. The most fundamental shape is `Curve`, which is just a collection of (x, y) coordinates in two arrays `x` and `y`. Drawing the `Curve` object is a matter of plotting `y` versus `x`. For class `Rectangle` the `x` and `y` arrays contain the corner points of the rectangle in counterclockwise direction, starting and ending with in the lower left corner.

Class `Line` is also a simple class:

```
class Line(Shape):
    def __init__(self, start, end):
        x = [start[0], end[0]]
        y = [start[1], end[1]]
        self.shapes = {'line': Curve(x, y)}
```

Here we only need two points, the start and end point on the line. However, we may want to add some useful functionality, e.g., the ability to give an x coordinate and have the class calculate the corresponding y coordinate:

```
    def __call__(self, x):
        """Given x, return y on the line."""
        x, y = self.shapes['line'].x, self.shapes['line'].y
        self.a = (y[1] - y[0])/(x[1] - x[0])
        self.b = y[0] - self.a*x[0]
        return self.a*x + self.b
```

Unfortunately, this is too simplistic because vertical lines cannot be handled (infinite `self.a`). The true source code of `Line` therefore provides a more general solution at the cost of significantly longer code with more tests.

A circle implies a somewhat increased complexity. Again we represent the geometric object by a `Curve` object, but this time the `Curve` object needs to store a large number of points on the curve such that a plotting program produces a visually smooth curve. The points on the circle must be calculated manually in the constructor of class `Circle`. The formulas for points (x, y) on a curve with radius R and center at (x_0, y_0) are given by

$$
\begin{aligned}
x &= x_0 + R\cos(t),\\
y &= y_0 + R\sin(t),
\end{aligned}
$$

where $t \in [0, 2\pi]$. A discrete set of t values in this interval gives the corresponding set of (x, y) coordinates on the circle. The user must specify the resolution as the number of t values. The circle's radius and center must of course also be specified.

We can write the `Circle` class as

```
class Circle(Shape):
    def __init__(self, center, radius, resolution=180):
        self.center, self.radius = center, radius
        self.resolution = resolution

        t = linspace(0, 2*pi, resolution+1)
        x0 = center[0];  y0 = center[1]
        R = radius
        x = x0 + R*cos(t)
        y = y0 + R*sin(t)
        self.shapes = {'circle': Curve(x, y)}
```

As in class `Line` we can offer the possibility to give an angle θ (equivalent to t in the formulas above) and then get the corresponding x and y coordinates:

```
    def __call__(self, theta):
        """Return (x, y) point corresponding to angle theta."""
        return self.center[0] + self.radius*cos(theta), \
               self.center[1] + self.radius*sin(theta)
```

There is one flaw with this method: it yields illegal values after a translation, scaling, or rotation of the circle.

A part of a circle, an arc, is a frequent geometric object when drawing mechanical systems. The arc is constructed much like a circle, but t runs in $[\theta_s, \theta_s + \theta_a]$. Giving θ_s and θ_a the slightly more descriptive names `start_angle` and `arc_angle`, the code looks like this:

```
class Arc(Shape):
    def __init__(self, center, radius,
                 start_angle, arc_angle,
                 resolution=180):
        self.start_angle = radians(start_angle)
        self.arc_angle = radians(arc_angle)

        t = linspace(self.start_angle,
                     self.start_angle + self.arc_angle,
                     resolution+1)
        x0 = center[0];  y0 = center[1]
        R = radius
```

```
        x = x0 + R*cos(t)
        y = y0 + R*sin(t)
        self.shapes = {'arc': Curve(x, y)}
```

Having the Arc class, a Circle can alternatively be defined as a subclass specializing the arc to a circle:

```
class Circle(Arc):
    def __init__(self, center, radius, resolution=180):
        Arc.__init__(self, center, radius, 0, 360, resolution)
```

Class curve. Class Curve sits on the coordinates to be drawn, but how is that done? The constructor of class Curve just stores the coordinates, while a method draw sends the coordinates to the plotting program to make a graph. Or more precisely, to avoid a lot of (e.g.) Matplotlib-specific plotting commands in class Curve we have created a small layer with a simple programming interface to plotting programs. This makes it straightforward to change from Matplotlib to another plotting program. The programming interface is represented by the drawing_tool object and has a few functions:

- plot_curve for sending a curve in terms of x and y coordinates to the plotting program,
- set_coordinate_system for specifying the graphics area,
- erase for deleting all elements of the graph,
- set_grid for turning on a grid (convenient while constructing the figure),
- set_instruction_file for creating a separate file with all plotting commands (Matplotlib commands in our case),
- a series of set_X functions where X is some property like linecolor, linestyle, linewidth, filled_curves.

This is basically all we need to communicate to a plotting program.

Any class in the Shape hierarchy inherits set_X functions for setting properties of curves. This information is propagated to all other shape objects in the self.shapes dictionary. Class Curve stores the line properties together with the coordinates of its curve and propagates this information to the plotting program. When saying vehicle.set_linewidth(10), all objects that make up the vehicle object will get a set_linewidth(10) call, but only the Curve object at the end of the chain will actually store the information and send it to the plotting program.

A rough sketch of class Curve reads

```
class Curve(Shape):
    """General curve as a sequence of (x,y) coordintes."""
    def __init__(self, x, y):
        self.x = asarray(x, dtype=float)
        self.y = asarray(y, dtype=float)
```

```
    def draw(self):
        drawing_tool.plot_curve(
            self.x, self.y,
            self.linestyle, self.linewidth, self.linecolor, ...)

    def set_linewidth(self, width):
        self.linewidth = width

    det set_linestyle(self, style):
        self.linestyle = style
    ...
```

Compound geometric objects. The simple classes `Line`, `Arc`, and `Circle` could can the geometric shape through just one `Curve` object. More complicated shapes are built from instances of various subclasses of `Shape`. Classes used for professional drawings soon get quite complex in composition and have a lot of geometric details, so here we prefer to make a very simple composition: the already drawn vehicle from Figure 2. That is, instead of composing the drawing in a Python program as shown above, we make a subclass `Vehicle0` in the `Shape` hierarchy for doing the same thing.

The `Shape` hierarchy is found in the `pysketcher` package, so to use these classes or derive a new one, we need to import `pysketcher`. The constructor of class `Vehicle0` performs approximately the same statements as in the example program we developed for making the drawing in Figure 2.

```
from pysketcher import *

class Vehicle0(Shape):
    def __init__(self, w_1, R, L, H):
        wheel1 = Circle(center=(w_1, R), radius=R)
        wheel2 = wheel1.copy()
        wheel2.translate((L,0))

        under = Rectangle(lower_left_corner=(w_1-2*R, 2*R),
                          width=2*R + L + 2*R, height=H)
        over  = Rectangle(lower_left_corner=(w_1, 2*R + H),
                          width=2.5*R, height=1.25*H)

        wheels = Composition(
            {'wheel1': wheel1, 'wheel2': wheel2})
        body = Composition(
            {'under': under, 'over': over})

        vehicle = Composition({'wheels': wheels, 'body': body})
        xmax = w_1 + 2*L + 3*R
        ground = Wall(x=[R, xmax], y=[0, 0], thickness=-0.3*R)

        self.shapes = {'vehicle': vehicle, 'ground': ground}
```

Any subclass of `Shape` *must* define the `shapes` attribute, otherwise the inherited `draw` method (and a lot of other methods too) will not work.

The painting of the vehicle, as shown in the right part of Figure 6, could in class `Vehicle0` be offered by a method:

```
    def colorful(self):
        wheels = self.shapes['vehicle']['wheels']
        wheels.set_filled_curves('blue')
        wheels.set_linewidth(6)
        wheels.set_linecolor('black')
        under = self.shapes['vehicle']['body']['under']
        under.set_filled_curves('red')
        over = self.shapes['vehicle']['body']['over']
        over.set_filled_curves(pattern='/')
        over.set_linewidth(14)
```

The usage of the class is simple: after having set up an appropriate coordinate system as previously shown, we can do

```
vehicle = Vehicle0(w_1, R, L, H)
vehicle.draw()
drawing_tool.display()
```

and go on the make a painted version by

```
drawing_tool.erase()
vehicle.colorful()
vehicle.draw()
drawing_tool.display()
```

A complete code defining and using class `Vehicle0` is found in the file `vehicle2.py`.

The `pysketcher` package contains a wide range of classes for various geometrical objects, particularly those that are frequently used in drawings of mechanical systems.

3.2 Adding functionality via recursion

The really powerful feature of our class hierarchy is that we can add much functionality to the superclass `Shape` and to the "bottom" class `Curve`, and then all other classes for various types of geometrical shapes immediately get the new functionality. To explain the idea we may look at the `draw` method, which all classes in the `Shape` hierarchy must have. The inner workings of the `draw` method explain the secrets of how a series of other useful operations on figures can be implemented.

Basic principles of recursion. Note that we work with two types of hierarchies in the present documentation: one Python *class hierarchy*, with `Shape` as superclass, and one *object hierarchy* of figure elements in a specific figure. A subclass of `Shape` stores its figure in the `self.shapes` dictionary. This dictionary represents the object hierarchy of figure elements for that class. We want to make one `draw` call for an instance, say our class `Vehicle0`, and then we want this call to be propagated to *all* objects that are contained in `self.shapes` and all is nested subdictionaries. How is this done?

The natural starting point is to call `draw` for each `Shape` object in the `self.shapes` dictionary:

```python
def draw(self):
    for shape in self.shapes:
        self.shapes[shape].draw()
```

This general method can be provided by class `Shape` and inherited in subclasses like `Vehicle0`. Let `v` be a `Vehicle0` instance. Seemingly, a call `v.draw()` just calls

```python
v.shapes['vehicle'].draw()
v.shapes['ground'].draw()
```

However, in the former call we call the `draw` method of a `Composition` object whose `self.shapes` attributed has two elements: `wheels` and `body`. Since class `Composition` inherits the same `draw` method, this method will run through `self.shapes` and call `wheels.draw()` and `body.draw()`. Now, the `wheels` object is also a `Composition` with the same `draw` method, which will run through `self.shapes`, now containing the `wheel1` and `wheel2` objects. The `wheel1` object is a `Circle`, so calling `wheel1.draw()` calls the `draw` method in class `Circle`, but this is the same `draw` method as shown above. This method will therefore traverse the circle's `shapes` dictionary, which we have seen consists of one `Curve` element.

The `Curve` object holds the coordinates to be plotted so here `draw` really needs to do something "physical", namely send the coordinates to the plotting program. The `draw` method is outlined in the short listing of class `Curve` shown previously.

We can go to any of the other shape objects that appear in the figure hierarchy and follow their `draw` calls in the similar way. Every time, a `draw` call will invoke a new `draw` call, until we eventually hit a `Curve` object at the "bottom" of the figure hierarchy, and then that part of the figure is really plotted (or more precisely, the coordinates are sent to a plotting program).

When a method calls itself, such as `draw` does, the calls are known as *recursive* and the programming principle is referred to as *recursion*. This technique is very often used to traverse hierarchical structures like the figure structures we work with here. Even though the hierarchy of objects building up a figure are of different types, they all inherit the same `draw` method and therefore exhibit the same behavior with respect to drawing. Only the `Curve` object has a different `draw` method, which does not lead to more recursion.

Explaining recursion. Understanding recursion is usually a challenge. To get a better idea of how recursion works, we have equipped class `Shape` with a method `recurse` that just visits all the objects in the `shapes` dictionary and prints out a message for each object. This feature allows us to trace the execution and see exactly where we are in the hierarchy and which objects that are visited.

The `recurse` method is very similar to `draw`:

```python
def recurse(self, name, indent=0):
    # print message where we are (name is where we come from)
    for shape in self.shapes:
        # print message about which object to visit
        self.shapes[shape].recurse(indent+2, shape)
```

The `indent` parameter governs how much the message from this `recurse` method is intended. We increase `indent` by 2 for every level in the hierarchy, i.e., every row of objects in Figure 8. This indentation makes it easy to see on the printout how far down in the hierarchy we are.

A typical message written by `recurse` when `name` is `'body'` and the `shapes` dictionary has the keys `'over'` and `'under'`, will be

```
      Composition: body.shapes has entries 'over', 'under'
      call body.shapes["over"].recurse("over", 6)
```

The number of leading blanks on each line corresponds to the value of `indent`. The code printing out such messages looks like

```
def recurse(self, name, indent=0):
    space = ' '*indent
    print space, '%s: %s.shapes has entries' % \
          (self.__class__.__name__, name), \
          str(list(self.shapes.keys()))[1:-1]

    for shape in self.shapes:
        print space,
        print 'call %s.shapes["%s"].recurse("%s", %d)' % \
              (name, shape, shape, indent+2)
        self.shapes[shape].recurse(shape, indent+2)
```

Let us follow a `v.recurse('vehicle')` call in detail, `v` being a `Vehicle0` instance. Before looking into the output from `recurse`, let us get an overview of the figure hierarchy in the `v` object (as produced by `print v`)

```
ground
    wall
vehicle
    body
        over
            rectangle
        under
            rectangle
    wheels
        wheel1
            arc
        wheel2
            arc
```

The `recurse` method performs the same kind of traversal of the hierarchy, but writes out and explains a lot more.

The data structure represented by `v.shapes` is known as a *tree*. As in physical trees, there is a *root*, here the `v.shapes` dictionary. A graphical illustration of the tree (upside down) is shown in Figure 8. From the root there are one or more branches, here two: `ground` and `vehicle`. Following the `vehicle` branch, it has two new branches, `body` and `wheels`. Relationships as in family trees are often used to describe the relations in object trees too: we say that `vehicle` is the parent of `body` and that `body` is a child of `vehicle`. The term *node* is also often used to describe an element in a tree. A node may have several other nodes as *descendants*.

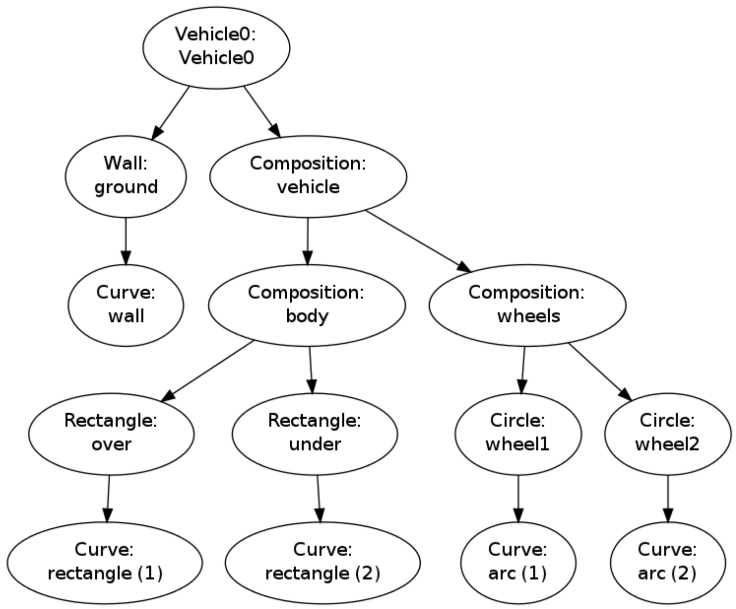


Figure 8: Hierarchy of figure elements in an instance of class `Vehicle0`.

Recursion is the principal programming technique to traverse tree structures. Any object in the tree can be viewed as a root of a subtree. For example, `wheels` is the root of a subtree that branches into `wheel1` and `wheel2`. So when processing an object in the tree, we imagine we process the root and then recurse into a subtree, but the first object we recurse into can be viewed as the root of the subtree, so the processing procedure of the parent object can be repeated.

A recommended next step is to simulate the `recurse` method by hand and carefully check that what happens in the visits to `recurse` is consistent with the output listed below. Although tedious, this is a major exercise that guaranteed will help to demystify recursion.

A part of the printout of `v.recurse('vehicle')` looks like

```
Vehicle0: vehicle.shapes has entries 'ground', 'vehicle'
call vehicle.shapes["ground"].recurse("ground", 2)
  Wall: ground.shapes has entries 'wall'
  call ground.shapes["wall"].recurse("wall", 4)
    reached "bottom" object Curve
call vehicle.shapes["vehicle"].recurse("vehicle", 2)
  Composition: vehicle.shapes has entries 'body', 'wheels'
  call vehicle.shapes["body"].recurse("body", 4)
    Composition: body.shapes has entries 'over', 'under'
    call body.shapes["over"].recurse("over", 6)
      Rectangle: over.shapes has entries 'rectangle'
      call over.shapes["rectangle"].recurse("rectangle", 8)
        reached "bottom" object Curve
    call body.shapes["under"].recurse("under", 6)
      Rectangle: under.shapes has entries 'rectangle'
```

```
        call under.shapes["rectangle"].recurse("rectangle", 8)
          reached "bottom" object Curve
...
```

This example should clearly demonstrate the principle that we can start at any object in the tree and do a recursive set of calls with that object as root.

3.3 Scaling, translating, and rotating a figure

With recursion, as explained in the previous section, we can within minutes equip *all* classes in the `Shape` hierarchy, both present and future ones, with the ability to scale the figure, translate it, or rotate it. This added functionality requires only a few lines of code.

Scaling. We start with the simplest of the three geometric transformations, namely scaling. For a `Curve` instance containing a set of n coordinates (x_i, y_i) that make up a curve, scaling by a factor a means that we multiply all the x and y coordinates by a:

$$x_i \leftarrow ax_i, \quad y_i \leftarrow ay_i, \quad i = 0, \ldots, n-1\,.$$

Here we apply the arrow as an assignment operator. The corresponding Python implementation in class `Curve` reads

```python
class Curve:
    ...
    def scale(self, factor):
        self.x = factor*self.x
        self.y = factor*self.y
```

Note here that `self.x` and `self.y` are Numerical Python arrays, so that multiplication by a scalar number `factor` is a vectorized operation.

An even more efficient implementation is to make use of in-place multiplication in the arrays,

```python
class Curve:
    ...
    def scale(self, factor):
        self.x *= factor
        self.y *= factor
```

as this saves the creation of temporary arrays like `factor*self.x`.

In an instance of a subclass of `Shape`, the meaning of a method `scale` is to run through all objects in the dictionary `shapes` and ask each object to scale itself. This is the same delegation of actions to subclass instances as we do in the `draw` (or `recurse`) method. All objects, except `Curve` instances, can share the same implementation of the `scale` method. Therefore, we place the `scale` method in the superclass `Shape` such that all subclasses inherit the method. Since `scale` and `draw` are so similar, we can easily implement the `scale` method in class `Shape` by copying and editing the `draw` method:

```python
class Shape:
    ...
    def scale(self, factor):
        for shape in self.shapes:
            self.shapes[shape].scale(factor)
```

This is all we have to do in order to equip all subclasses of `Shape` with scaling functionality! Any piece of the figure will scale itself, in the same manner as it can draw itself.

Translation. A set of coordinates (x_i, y_i) can be translated v_0 units in the x direction and v_1 units in the y direction using the formulas

$$x_i \leftarrow x_i + v_0, \quad y_i \leftarrow y_i + v_1, \quad i = 0, \ldots, n-1\,.$$

The natural specification of the translation is in terms of the vector $v = (v_0, v_1)$. The corresponding Python implementation in class `Curve` becomes

```python
class Curve:
    ...
    def translate(self, v):
        self.x += v[0]
        self.y += v[1]
```

The translation operation for a shape object is very similar to the scaling and drawing operations. This means that we can implement a common method `translate` in the superclass `Shape`. The code is parallel to the `scale` method:

```python
class Shape:
    ....
    def translate(self, v):
        for shape in self.shapes:
            self.shapes[shape].translate(v)
```

Rotation. Rotating a figure is more complicated than scaling and translating. A counter clockwise rotation of θ degrees for a set of coordinates (x_i, y_i) is given by

$$\begin{aligned}
\bar{x}_i &\leftarrow x_i \cos\theta - y_i \sin\theta,\\
\bar{y}_i &\leftarrow x_i \sin\theta + y_i \cos\theta\,.
\end{aligned}$$

This rotation is performed around the origin. If we want the figure to be rotated with respect to a general point (x, y), we need to extend the formulas above:

$$\begin{aligned}
\bar{x}_i &\leftarrow x + (x_i - x)\cos\theta - (y_i - y)\sin\theta,\\
\bar{y}_i &\leftarrow y + (x_i - x)\sin\theta + (y_i - y)\cos\theta\,.
\end{aligned}$$

The Python implementation in class `Curve`, assuming that θ is given in degrees and not in radians, becomes

```
def rotate(self, angle, center):
    angle = radians(angle)
    x, y = center
    c = cos(angle);  s = sin(angle)
    xnew = x + (self.x - x)*c - (self.y - y)*s
    ynew = y + (self.x - x)*s + (self.y - y)*c
    self.x = xnew
    self.y = ynew
```

The `rotate` method in class `Shape` follows the principle of the `draw`, `scale`, and `translate` methods.

We have already seen the `rotate` method in action when animating the rolling wheel at the end of Section 1.1.

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